TEXTURAL FRACTOGRAPHY: APPLICATION OF GIBBS RANDOM FIELDS

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ABSTRACT

Within the textural fractography of fatigue failures, the correspondence between the velocity of crack growth and the texture in images of crack surface is investigated. A Gibbs random field (GRF) model considering pairwise interaction was used to describe the texture. The structure of the model and the process of estimating its parameters (potential) are described. A multilinear regression was proposed for relating parameters of GRF to the crack velocity. The theory was applied on results of four fatigue experiments in specimens from stainless steel AISI 304L used in nuclear power industry. The theory of GRF has been proved to be a powerful instrument for the textural fractography.

KEYWORDS

fractography, fatigue, image, texture, Gibbs random field, stochastic relaxation

NOMENCLATURE

Gibbs random fields

GRF	Gibbs random field
IRF	independent random field
$\boldsymbol{x} \in \boldsymbol{X}$, $\boldsymbol{x}_o$	image, training image
$r = 1,\ldots,M$, $c = 1,\ldots,N$	indexing pixels of an image
$x(r,c) = u$, $u \in \{0,1,\ldots, n_u\text{-}1\} = U$	gray level in a pixel
n_u	number of gray levels
$d = u_1 - u_2$, $d \in \{-(n_u\text{-}1),\ldots, n_u\text{-}1\} = D$	interaction of gray levels in a pair of pixels
n_d	number of interaction levels
$\boldsymbol{W}$	search window
$[i,j] \in \boldsymbol{W}$	clique, set of pixel pairs satisfying $r_1\text{-}r_2 = i$, $c_1\text{-}c_2 = j$
$\rho_{i,j}$	relative size of the clique $[i,j]$
$\boldsymbol{h}(\boldsymbol{x}) = \{h_{i,j,d}(\boldsymbol{x})\}$	gray level co-occurrence histogram of image $\boldsymbol{x}$
$\boldsymbol{f}(\boldsymbol{x}) = \{f_{i,j,d}(\boldsymbol{x})\}$	relative frequency of interactions of image $\boldsymbol{x}$
$\boldsymbol{V} = \{V_{i,j,d}\}$	potential
$H(\boldsymbol{x})$	Hamiltonian of image $\boldsymbol{x}$
$L(\boldsymbol{V}, \boldsymbol{x})$	log-likelihood function of potential $\boldsymbol{V}$ at image $\boldsymbol{x}$
$\boldsymbol{e} = \{e_{i,j}\}$	relative energy of the clique $[i,j]$

Fatigue

CGR = v [μm/cycle], $y = \log_{10}(v)$	velocity of crack growth (crack growth rate)

INTRODUCTION

The quantitative microfractography of fatigue failures is concerned especially with the reconstitution of the history of a fatigue crack growth process. Within laboratory tests, specimens of the material are loaded under service conditions whereas the crack growth process is recorded. Fracture surfaces are documented by SEM and images are studied to relate some information present in the morphology of the crack surface to the macroscopic crack velocity (crack growth rate - CGR). So a basis is obtained on which an unknown CGR can be estimated from fracture surfaces of real parts. Finally, the crack growth process is reconstituted using integration of CGR along the crack growth direction.

The traditional method is based on fractographic features - strictly defined measurable objects in the morphology of the fracture surface. For the case of fatigue, a typical feature is striations, fine parallel grooves in the fracture surface. The method cannot be used when striations are not visible, typically due to corrosion.

As an alternative, the textural method has been developed in our department since about 1990. A texture in image sense is a random structure of similar elements with some kind of ordering. In many cases, structures in images of fracture surfaces can be studied as image textures. The main problem in fractography consists in the continuous brightness scale and absence of distinct borders of textural elements.

For the application of the textural method, especially suitable is the mezoscopic dimensional area with SEM magnifications between macro- and microfractography (about $30 \div 500$ x). These magnifications were not used very much in the past for the absence of measurable objects in images. A particular setting of the magnification is limited by several conditions related to individual images, to the whole set of images and to image discretization. Images must be pre-processed [3] to obtain a homogeneous texture which is proper for the analysis.

In the textural method, fractographic information is extracted in the form of integral parameters of the whole image. Several routine analytical procedures have been developed within two general approaches:

- computing statistical parameters directly from gray-scale images,
- extraction of textural elements followed by the application of binary random field models.

Within the first approach, correlation and spectral parameters [2] have been used till now. In the present contribution, modelling the texture as a Gibbs random field will be applied.

THEORY

Model of a random field [1]

In our narrowed sense, random field will be a population $\boldsymbol{X}$ of images $\boldsymbol{x}$ of the size M x N, with joined probabilities $\Pr(\boldsymbol{x})$

$$\forall\, \boldsymbol{x} \in \boldsymbol{X} \quad \Pr(\boldsymbol{x}) > 0, \ \sum_{\boldsymbol{x} \in \boldsymbol{X}} \Pr(\boldsymbol{x}) = 1 \quad . \tag{1}$$

A general model of an image random field can be characterized by properties of selected sets of pixels, refered to as cliques. In the present application, the structure will be determined by

- brightness level in single pixels,
- a relationship between brightness of two pixels, denoted as *pairwise interaction*.

It means single pixels and pairs of pixels with a general mutual position are taken for cliques.

A presumption simplifying significantly the model is that the texture is homogeneous. It is a generalization of the property of homogeneity of one-dimensional random sequences. The texture may be anisotropic, but properties along a given direction are not dependent on the localization within the image. In a homogeneous texture, all pairs of pixels having an identical relative position are equivalent. Therefore, pair cliques will be defined by the differences of indices, $i = r_1 - r_2$, $j = c_1 - c_2$, and will be denoted as $[i,j]$. Cliques can be represented by a *search window* ***W***. Fig.1 illustrates assigning pairs of pixels into ***W***. Single pixels create a special clique characterized by the distance vector [0,0].

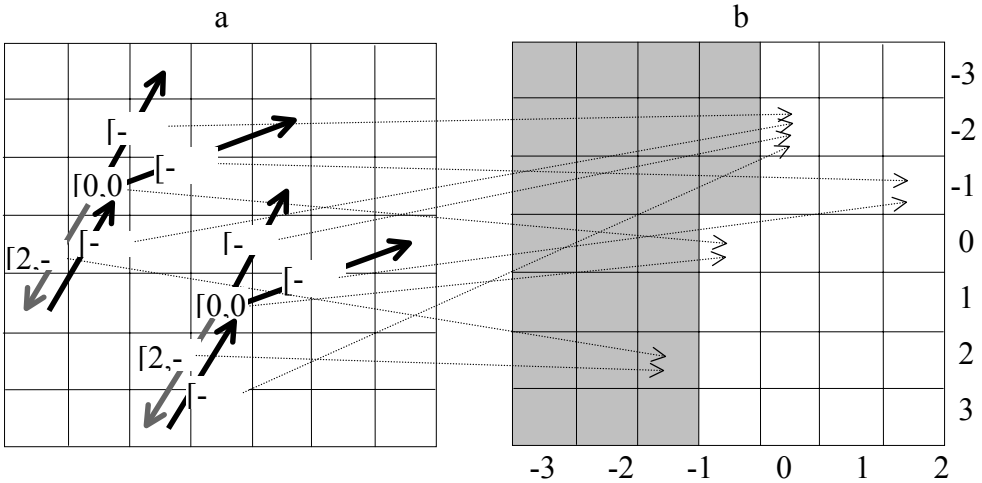


Fig.1: Organizing pairs of pixels (a) into a search window (b). The center [0,0] represents the clique of single pixels.

The whole search window contains each clique twice as $[i,j]$ and $[-i,-j]$, through two elements placed in the search window symmetrically to the center. Therefore we use only one half of it, the other being shaded.

Because a digital image is of a finite size, different cliques have different cardinalities. Clique $[i, j] \in \boldsymbol{W}$ in an image of size $M \times N$ contains $(M-|i|)\cdot(N-|j|)$ pairs. Then *relative size of a clique* $\rho_{i,j}$ is

$$\rho_{i,j} = \frac{(M-|i|)(N-|j|)}{MN} \quad . \tag{2}$$

In general, the search window ***W*** involves all cliques that can be taken into account within the image. However, when the texture does not contain a deterministically periodic component, the significance of cliques decreases with an increasing distance of both pixels within the pair. In terms of the search window ***W*,** the significance decreases with an increasing distance from the center. Therefore the search window ***W*** can be reduced to the central part containing the set of significant cliques.

Let us denote u=0,1,…,n_u-1 gray levels of single pixels, n_u the number of gray levels within the given scale and U the set of them. Similarly, $d = d(u_1, u_2)$ will be the interaction of two pixels with gray levels u_1 and u_2, n_d the number of all possible interactions and D the set of interactions. In the simpliest variant of the model, interactions are defined by the difference

between gray levels in the two pixels of a pair. They can be denoted by index $d = u_1 - u_2$, acquiring values of $d = -(n_u-1),\ldots,n_u-1$. The number of possible interactions is $n_d = 2n_u-1$.

For clique [0,0] of single pixels, index $d = u$ equal to gray level is to be used. In the following text, this case will be understood as a unitary interaction of a pixel with itself, and will be included among all interactions $d \in D$.

The information about numbers of gray level combinations in cliques is given by a gray level co-occurrence histogram, which will be represented by a three-dimensional array $\boldsymbol{h}$. $h_{i,j,d}$ is the number of gray level combinations $d \in D$ in a clique $[i, j] \in \boldsymbol{W}$. In formal relations all the three indices are defined by their natural values including negative ones, while for computational implementation their ranges have to be shifted. For single pixel clique $[0, 0] \in \boldsymbol{W}$ only $n_u < n_d$ values of $\boldsymbol{h}$ are present in the third dimension, the remaining values being defined by zeros.

The algorithm for creating histogram $\boldsymbol{h}$ consists in moving search window $\boldsymbol{W}$ pixelwise through the image. In every position, for all combinations of indices $[i,j]$ pairs of pixels are selected according to the searching rule. For every pair, the combination of gray levels in both pixels is recomputed to interaction d, and the value $h_{i,j,d}$ increased by one.

Later we will also need *relative frequency of interactions* $\boldsymbol{f}$ which is defined

$$f_{i,j,d} = \frac{h_{i,j,d}}{(M-|i|)(N-|j|)} \quad . \tag{3}$$

$f_{i,j,d}$ is an estimate of apriori probability of occurrence of an interaction d in a clique $[i, j]$. The pertinence of quantities to an image $\boldsymbol{x}$ will be denoted as $\boldsymbol{h}(\boldsymbol{x}), \boldsymbol{f}(\boldsymbol{x})$, etc.

Gibbs probability distribution

A Gibbs random field (GRF) [1] is a model which originated in statistical physics. It describes equilibrium probability distributions of large systems of interacting particles. While physical interactions are based on physical forces, in our case we know only interactions between two pixels, defined by conditional probabilities of gray levels. In the image analysis, terms *interaction* and *energy* are used as an analogy.

Energy is the basic concept in the Gibbs model. The total energy of an image $\boldsymbol{x}$ is determined by Hamiltonian $H(\boldsymbol{x})$ which depends on interactions - combinations of gray levels in cliques. A potential of gray level combinations is assigned to every clique. Potential values are represented by a three-dimensional array $\boldsymbol{V}$. The structure of $\boldsymbol{V}$ is similar to the structure of $\boldsymbol{h}$. $\boldsymbol{V}_{i,j,d}$ denotes a potential value belonging to clique $[i, j] \in \boldsymbol{W}$, and a gray level combination $d \in D$. Similarly to the case of $\boldsymbol{h}$ for [0,0] only $\boldsymbol{V}_{0,0,d}$, $d \in \{0,\ldots,n_u-1\}$, are defined while the remaining elements are zero. Hamiltonian is given by

$$H(\boldsymbol{x}) = \sum_{[i,j]\in \boldsymbol{W}} \sum_{d\in D} V_{i,j,d}\ h_{i,j,d}(\boldsymbol{x}) = \boldsymbol{V} * \boldsymbol{h}(\boldsymbol{x}) \ , \tag{4}$$

where * denotes a scalar product of arrays $\boldsymbol{V}$ and $\boldsymbol{h}(\boldsymbol{x})$ reshaped into vectors. How the potential $\boldsymbol{V}$ is estimated will be shown later.

Gibbs probability distribution (probability of image $\boldsymbol{x}_o$ within the space of all realizations of the given GRF with potential $\boldsymbol{V}$) is given by

$$\Pr(\boldsymbol{x}_o) = \frac{\exp\left(-H(\boldsymbol{x}_o)\right)}{\sum_{\boldsymbol{x}\in\boldsymbol{X}} \exp\left(-H(\boldsymbol{x})\right)} = \frac{\exp\left(-\boldsymbol{V} * \boldsymbol{h}(\boldsymbol{x}_o)\right)}{\sum_{\boldsymbol{x}\in\boldsymbol{X}} \exp\left(-\boldsymbol{V} * \boldsymbol{h}(\boldsymbol{x})\right)} \quad . \tag{5}$$

Pixelwise stochastic relaxation
is a method [1] of generating samples of a GRF as a *Markov chain*. Given a potential $\boldsymbol{V}$, each next sample is obtained by random choosing according to a conditional probability given only by the current sample. The chain begins with an initial image $\boldsymbol{x}^{(0)}$. In every *microstep*, the gray level in a randomly chosen pixel $[r,c]$ is changed to a new random value generated from the distribution conditional on current gray levels in a set of pixels interacting with $[r,c]$. A *macrostep* is a set of microsteps in which all pixels of the image have been passed.

In our case, the set of interacting pixels is defined by the search window $\boldsymbol{W}$, excluding clique [0,0] denoting the pixel $[r,c]$ itself. It can be described as $\{ [r+i, c+j], [i,j]\in \boldsymbol{W} \setminus [0,0] \}$. From relation (5) it follows that conditional probability of the gray level in the given pixel $[r,c]$ provided values of gray levels in the set of interacting pixels is

$$\Pr\left(x(r,c)\middle|x(r+i,c+j)\ ,[i,j]\in \boldsymbol{W}\setminus[0,0]\right) = \frac{\exp \sum_{[i,j]\in \boldsymbol{W}\setminus[0,0]} \boldsymbol{V}_{i,j,x(r,c)-x(r+i,c+j)}}{\sum_{u\in U} \exp \sum_{[i,j]\in \boldsymbol{W}\setminus[0,0]} \boldsymbol{V}_{i,j,u-x(r+i,c+j)}} \quad . \tag{6}$$

The set of probabilities $\Pr\left(\boldsymbol{x}(r,c) \le u\right)$, $u = 0,1,\ldots,n_u\text{-}1$ follows as

$$\Pr\left(x(r,c) \le u\right) = \sum_{b=0}^{u} \left(\Pr\left(x(r,c)\right) = b\middle|\ x(r+i,c+j),\ [i,j]\in \boldsymbol{W}\setminus[0,0]\right)\right),\ u = 0,1,\ldots,n_u\text{-}1\ . \tag{7}$$

After generating a uniform probability P, the value u corresponding to the nearest greater probability of the set (7) is selected.

For pixels within marginal strips of an image, the set of interacting pixels is not complete - interactions range beyond the image. A possibility how to solve this problem is to repeat the image periodically. Values from the opposite side of the image are taken for the ones that are missing beyond the margin. Then, if we put copies of a simulated image as tiles, we can see that the texture joins continually at borders.

Estimating of potential V
The solution follows two steps [1]: a first approximation derived from truncated Taylor expansion of the log-likelihood function will be refined by an algorithm using stochastic relaxation.

Maximum likelihood estimate
To estimate a potential $\boldsymbol{V}$ from a given GRF *training sample* $\boldsymbol{x}_o$, maximizing of the likelihood function can be applied. The likelihood function gives a probability of generating the sample $\boldsymbol{x}_o$ under the condition of a potential $\boldsymbol{V}$. For computational reasons, logarithm of the likelihood function is used. After substituting from (5) we receive

$$L\left(\boldsymbol{V},\boldsymbol{x}_o\right) = \frac{1}{MN}\log \Pr(\boldsymbol{x}_o|\boldsymbol{V}) = \frac{1}{MN}\left(\boldsymbol{V} * \boldsymbol{h}(\boldsymbol{x}_o) - \log \sum_{\boldsymbol{x}\in\boldsymbol{X}} \exp\left(\boldsymbol{V} * \boldsymbol{h}(\boldsymbol{x})\right)\right) \quad . \tag{8}$$

First and second derivatives are

$$\frac{\partial L(\boldsymbol{V},\boldsymbol{x}_o)}{\partial \boldsymbol{V}} = \frac{1}{MN}\left(\boldsymbol{h}(\boldsymbol{x}_o) - \underset{\boldsymbol{x}\in \boldsymbol{X}|\boldsymbol{V}}{E}\left(\boldsymbol{h}(\boldsymbol{x})\right)\right) \ , \tag{9}$$

$$\frac{\partial^2 L(\boldsymbol{V},\boldsymbol{x}_o)}{\partial \boldsymbol{V}^2} = -\frac{1}{(MN)^2}\underset{\boldsymbol{x}\in \boldsymbol{X}|\boldsymbol{V}}{Cov}\left(\boldsymbol{h}(\boldsymbol{x})\right) \ , \tag{10}$$

where $E(..)$ denotes the mean value and $Cov(..)$ covariance of the histogram $\boldsymbol{h}$ over the random field (population of images) defined by the potential $\boldsymbol{V}$.

The condition of the zero first derivative means that given an estimate $\boldsymbol{V}$, the mean value of the histogram is equal to that of sample $\boldsymbol{x}_o$

$$\underset{\boldsymbol{x}\in \boldsymbol{X}|\boldsymbol{V}}{E}\left(\boldsymbol{h}(\boldsymbol{x})\right) = \boldsymbol{h}(\boldsymbol{x}_o) \ . \tag{11}$$

First approximation

The first approximation of the potential is based on a decomposition of the likelihood function into a truncated Taylor expansion about a point of zero potential $\boldsymbol{V} = \boldsymbol{0}$. Within the limited space of this contribution only main results may be shown.

The zero potential defines a random field with independent components (independent random field - IRF). In this case, probability of gray levels in a pixel is uniform and gray levels in different pixels are mutually independent. Probability of a gray level difference d in a double-pixel clique $[i,j]$ is (the fact that it is not a sample but a population characteristic is denoted by a bar)

$$\bar{f}_{i,j,d}(IRF) = \frac{n_u - |d|}{n_u^2} \quad \forall\ d \in D, \quad [i,j] \in \boldsymbol{W} \setminus [0,0] \ . \tag{12}$$

To hold compatibility with our convention, we will define $\bar{\boldsymbol{f}}_{0,0,d}(IRF)$ by the probability of a gray level - constant $1/n_u$ in the range of values $d = u = 0,1,\ldots,n_u\text{-}1$, and with zeros for remaining values. Then

$$\bar{f}_{0,0,d}(IRF) = \left\langle \begin{array}{ll} 1/n_u & \text{for } d = 1,\ldots,n_u \ , \\ 0 & \text{for } d = n_u+1,\ldots,n_d \ . \end{array} \right. \tag{13}$$

The first approximation of the potential is being sought for in the form with free parameter λ

$$V^{(0)}_{i,j,d} = \lambda\, \rho_{i,j}\left(f_{i,j,d}(\boldsymbol{x}_o) - \bar{f}_{i,j,d}(IRF)\right) \ , \quad [i,j] \in \boldsymbol{W} \ , \ d \in D \ . \tag{14}$$

The estimate of λ maximizing the value of the likelihood function is given by

$$\hat{\lambda} = \frac{\sum\limits_{[i,j]\in \boldsymbol{W}} \rho^2_{i,j} \cdot \sum\limits_{d\in D}\left(f_{i,j,d}(\boldsymbol{x}_o) - \bar{f}_{i,j,d}(IRF)\right)^2}{\sum\limits_{[i,j]\in \boldsymbol{W}} \rho^3_{i,j} \cdot \sum\limits_{d\in D} \bar{f}_{i,j,d}(IRF)\cdot\left(1 - \bar{f}_{i,j,d}(IRF)\right)\cdot\left(f_{i,j,d}(\boldsymbol{x}_o) - \bar{f}_{i,j,d}(IRF)\right)^2} \ . \tag{15}$$

The last operation of this step is reducing the search window $\boldsymbol{W}$ only to cliques that contain a significant interaction. The reduction is necessary because the computational complexity of the next step depends on cardinality of the set $\boldsymbol{W}$. The significance of cliques can be compared by their relative energies defined as

$$e_{i,j} = \sum_{d \in D} V_{i,j,d} \; f_{i,j,d}(\boldsymbol{x_o}) \; , \; [i,j] \in \boldsymbol{W} \; . \tag{16}$$

An example of matrix $\boldsymbol{e}$ is displayed in Fig.2. The values of relative energy of the most important cliques create a peak. The search window $\boldsymbol{W}$ may be limited to the area covering the region of the peak.

Stochastic refining

In order to refine the estimate of potential $\boldsymbol{V}$, a pixelwise stochastic relaxation may be used. In every macrostep, the potential is modified according to the difference between relative frequencies of interactions in the training image $\boldsymbol{x_o}$ and the last generated image $\boldsymbol{x}^{(t)}$

$$V_{i,j,d}^{(t+1)} = V_{i,j,d}^{(t)} + \beta(t)\rho_{i,j}\left(f_{i,j,d}(\boldsymbol{x_o}) - f_{i,j,d}(\boldsymbol{x}^{(t)})\right), \quad [i,j] \in \boldsymbol{W}, d \in D, \tag{17}$$

where $\boldsymbol{x}^{(t)}$ is a sample generated with current potential $\boldsymbol{V}^{(t)}$ from sample $\boldsymbol{x}^{(t-1)}$.

The chain starts with a sample $\boldsymbol{x}^{(0)}$ generated from a sample of IRF with the first potential approximation $\boldsymbol{V}^{(0)}$. The strength of the correction $\beta(t)$ should decrease with the number of steps passed. The form of

$$\beta(t) = 1/(c_1 + c_2 t) \tag{18}$$

has been used with empirically estimated values of parameters c_1 and c_2.

Difference between the training and current sample can be measured by chi-square distance

$$\Delta\left(\boldsymbol{x_o}, \boldsymbol{x}^{(t)}\right) = \sum_{[i,j] \in \boldsymbol{W}} \sum_{d \in \boldsymbol{D}} \left(f_{i,j,d}(\boldsymbol{x_o}) - f_{i,j,d}(\boldsymbol{x}^{(t)})\right)^2 . \tag{19}$$

When the chain tends toward the equilibrium, the difference decreases to a constant and its fluctuations decrease to zero. A comparison of the input image and the GRF sample after stochastic relaxation is shown in Fig.3.

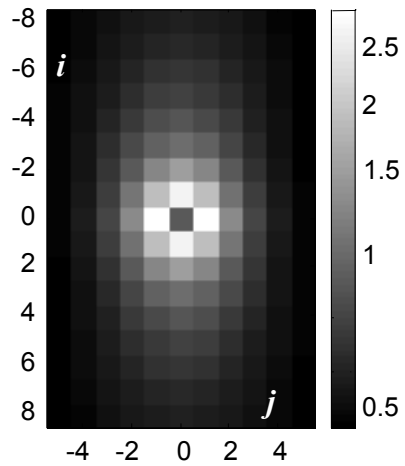
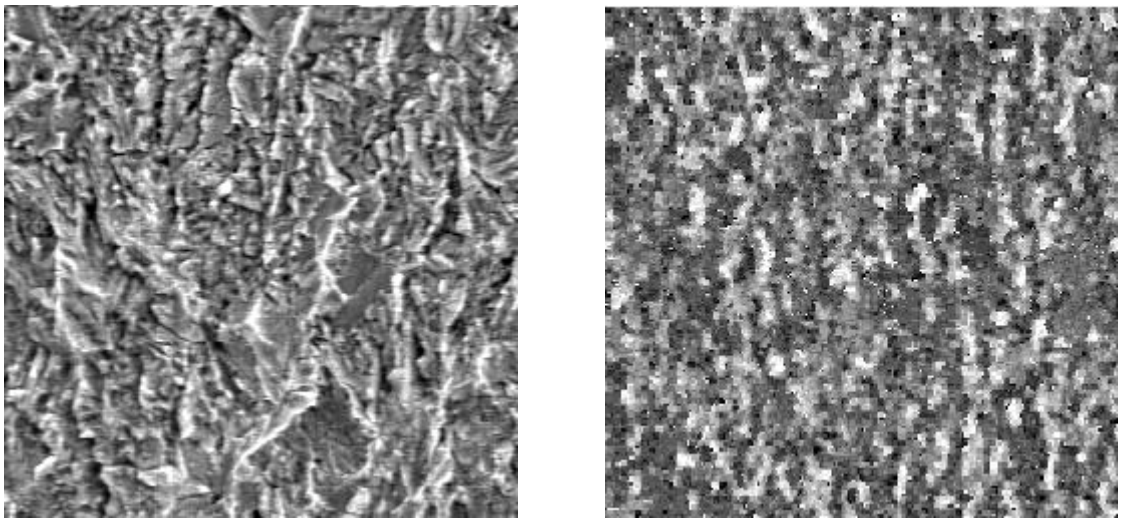


Fig.2: The significant part of the relative energy $\boldsymbol{e}$ (completed with respect to symmetry).



a 10μ b

Fig.3: Comparison of a - an input texture (detail 256 x 256 pixels),
b - sample of GRF model, potential estimated by stochastic relaxation (137 macrosteps) .

The multilinear model [2]

Our aim is to relate the characteristics of the GRF model of the image to the value of the crack velocity CGR = v. Due to general qualities of CGR its logarithm $y = \log_{10}(v)$ must be consi-dered. To characterise the GRF model, we have chosen a set of significant values of relative energy $e_{i,j}$ (16). For this application they are reshaped into a vector $\vec{\boldsymbol{e}} = \{e_a\}$, $a = 1,2,\ldots,k$. The simplest model expressing the CGR as a func-tion of a set of image parameters e_a is a multilinear function resulting into a regression equation (20).

$$\bar{y} = \sum_{a=1}^{k} b_a e_a + b_{k+1} \tag{20}$$

$$\boldsymbol{F} = \begin{bmatrix} e_1^1 & e_2^1 & \cdots & e_k^1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e_1^q & e_2^q & \cdots & e_k^q & 1 \end{bmatrix} \tag{21}$$

The values of parameters b can be estimated using the least squares method. The input information is composed of the set of images with assigned values of the CGR. From every image, a set of image GRF characteristics e_a is computed, completed with constant 1 and arranged into one row of matrix $\boldsymbol{F}$ (21). Then the system of equations can be written in the form (22) where $\boldsymbol{Y}$ is a column vector of logarithms of CGR assigned to single images and $\boldsymbol{B}$ (23) is a column vector of estimated parameters.

$$\bar{\boldsymbol{Y}} = \boldsymbol{FB} \tag{22}$$

$$\boldsymbol{B} = \begin{bmatrix} b_1 & b_2 & \ldots & b_k & b_{k+1} \end{bmatrix}' \tag{23}$$

Not all characteristics e_a predicate the CGR. An instrument for testing the significance is the test of the zero value of the estimated coefficients b_a, $a = 0,1,\ldots,k$. We test the hypothesis H_0: $b_a = 0$ against the alternative H_1: $b_a \neq 0$. The test criterion is a Student's t-distributed statistic (24). If the absolute value of t_a is lower than the critical value at the selected level of significance α and q-k-1 degrees

$$t_a = \frac{b_a}{s(b_a)} \tag{24}$$

$$s(b_a) = \sqrt{\left(\boldsymbol{s}_b^2\right)_{a,a}} \tag{24a}$$

$$\boldsymbol{s}_b^2 = s^2(\boldsymbol{F}'\boldsymbol{F})^{-1} \tag{24b}$$

$$s^2 = \frac{1}{q-u-1}\sum_{a=1}^{k}\left(y_a - \bar{y}\right)^2 \tag{24c}$$

$$\left|t_a\right| < t_{1-\alpha/2}(q-k-1) \tag{25}$$

of freedom (25), the hypothesis H_0 cannot be rejected and the a-th column in the matrix $\boldsymbol{F}$ should be excluded.

APPLICATION

The method was applied to a set of fatigue fracture surfaces of four labotarory specimens (C16÷19) of stainless steel AISI 304L used in nuclear power plants. The specimen type was CT (Fig.4) with the initial notch length 12.5 mm. Constant cycle loading with parameters $\Delta F = 3400$ N, $R = 0.3$, $f = 1$ Hz occured in water at 20°C. Crack length was measured by COD.

Fatigue crack surfaces were documented using SEM with magnification 200x. The sequence of images was located in the middle of the crack surface (Fig.4) and the images were distanced by 0.4 mm. The direction of the crack growth in images is bottom-up. The real area of one image is about 0.6 x 0.45 mm (the images overlap by 0.05 mm). The digital representation in 1200 x 1600 pixels and 256 brightness values was used. Images were pre-processed using normalisation [3] to exclude fluctuations in the mean brigtness and contrast. The whole number of images was 164. An example of a typical texture is shown in Fig.3a.

From frequently repeated records of the crack length vs. number of cycles the estimates of the CGR were computed. The course of the CGR related to the crack length was estimated and every image was assessed a value of the CGR pertinent to its middle.

Relative energies (16) have been computed for all images. For the multilinear model, 31 cliques $[i,j]$ significant in all images were chosen from the area of the search window. From them 25 have been found to be significant in multilinear regression (20) at the level of significance $\alpha = 0.05$. The result is plotted in Fig.5. The agreement between input velocities of the crack growth and that ones given back from the GRF model is very good.

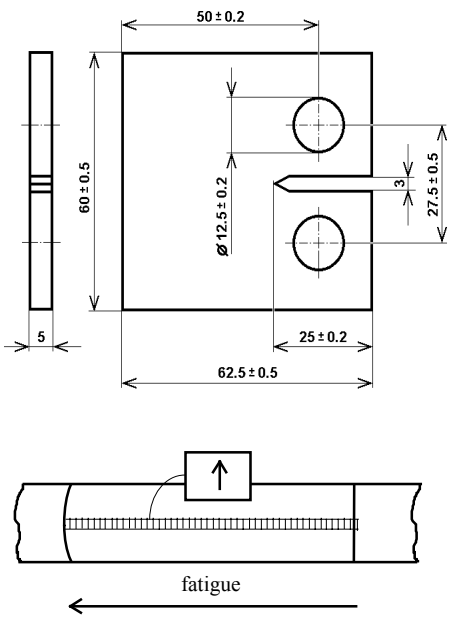


Fig. 4: Specimen for fatigue tests and locating of images in fatigue crack surface.

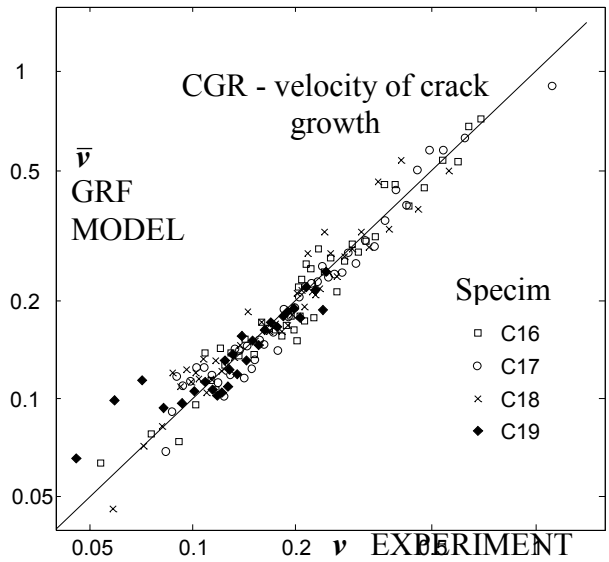


Fig. 5: Comparison of crack velocities v taken for input and $\bar{v}$ given back from the GRF model of image texture.
One point represents one image. The line $\bar{v} = v$ indicates the ideal agreement.

CONCLUSION

The theory of Gibbs random fields was applied in the simplest possible version. However, it reflected sensitively the relation between the velocity of a fatigue crack growth and the texture in images of crack surface. GRF model brings a new hopeful method for textural fractography.

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