ESTIMATION OF THE CRITICAL STRESS FOR FAILURE OF A PROTECTIVE LAYER

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ABCTRACT

This contribution describes a study of the behavior of a crack growing in a protective layer and penetrating through an interface into a substrate. Special attention is devoted to a crack touching the interface. Conditions for the stability of a crack terminating at the interface between two materials are analyzed. The problem is studied under the assumptions of linear-elastic fracture mechanics. A new tentative criterion of stability based on application of the strain energy density concept is formulated and applied to the problem. The suggested criterion is used to estimate the critical applied stress for the failure of a substrate caused by a crack growing through the protective layer. It is shown that the critical stress may strongly depend on the fracture toughness of the substrate and on the bi-material parameters.

KEYWORDS

Interface, protective layer, crack stability, critical stress.

INTRODUCTION

Components with protective coatings are increasingly being applied in engineering. The interface between two different materials (i.e., coating and substrate) represents a weak point for many applications of such structures. Generally, the existence of two regions with different material properties and the presence of an interface strongly influence the distribution of the stress in composite bodies. Fracture usually starts at a defect in the interface, and the complex nature of the stress in the vicinity of an interface influences the behavior of a crack.

This contribution describes a study of the behavior of a crack growing in a protective layer and penetrating through an interface and into a substrate. Special attention is devoted to a crack touching the interface. An example of such a configuration of a crack is the case of a brittle coating on a tough substrate, where a whole network of cracks very often appears and the individual cracks stop when they reach the interface. The aim of this contribution is to suggest a criterion which makes it possible to estimate the critical applied stress for the failure of such a substrate.

The behavior of stress singularities at the tip of a crack terminating at an interface between two different elastic materials has been studied [3,7,8], and a comprehensive theoretical treatment of the corresponding boundary problem exists in the literature. The formulas that describe the displacement and the distribution of stress in the vicinity of a crack terminating at the interface have long been known [15]. The common approach employed in the literature assumes the stress singularities to be of the form r^{-p}, where $0 < Re(p) < 1$ and r is the distance from the tip of the crack. The characteristic equation for obtaining the power of the singularities p can then be formulated by applying the corresponding boundary conditions.

Expressing the elastic properties of two different materials requires four constants, namely, the elastic modulus and the Poisson's ratio for both materials. If the two materials are strongly bonded together (i.e., interface of a welded type), the strains in the interface area depend on only two parameters only. In the following composite parameters α and β are used for plane strain

$$\alpha = \frac{\frac{E_1}{E_2} \cdot \frac{1+\nu_1}{1+\nu_2} - 1}{4(1-\nu_1)}, \qquad \beta = \frac{E_1}{E_2} \cdot \frac{1-\nu_2^2}{1-\nu_1^2} \tag{1}$$

where E_i and ν_i ($i = 1,2$) are the elastic modulus and the Poisson's ratio of the materials. The values of the power of singularity are then functions of both composite parameters, i.e., $p = p(\alpha, \beta)$. The stress distribution around the crack tip can thus be expressed as the sum of the terms

$$\sigma_{ij} = H_I / \sqrt{2\pi} f_{ij}(\phi, p, \alpha, \beta, \theta) / r^p \tag{2}$$

where $f_{ij} = f_{ij}(\theta, \alpha, \beta, \phi, p)$ is a known function of the polar angle θ and the two composite parameters. H_I is the value of the generalized stress intensity factor corresponding to the power of singularity p and must be determined numerically.

The fact that the value of the power of singularity p differs from 1/2 means that the conventional fracture mechanics approach (developed for homogeneous bodies where $p = 1/2$) cannot be applied.

The conditions necessary for the stability of a crack terminating at the interface between two materials are analyzed in this paper. The problem is studied under the assumptions of linear-elastic fracture mechanics, and a bi-material body with a crack terminating at the interface is used to model the configuration studied. A new tentative criterion of stability based on application of the strain energy density concept is then formulated. To this aim general formulas expressing the dependence of the strain energy density on the material characteristics α and β are introduced, and the criterion of stability for a crack with its tip at the interface is formulated. The suggested criterion is used to estimate the critical applied stress for the failure of a substrate caused by a crack growing through the protective layer. A numerical example showing the application of the stability criterion is presented.

STABILITY CONDITIONS

In the following chapter the stability conditions are formulated for a crack terminating at the interface at an angle ϕ, see Fig.1. Note that the crack generally propagates under mixed mode loading conditions.

Generalized strain energy density factor

The strain energy density factor S was originally introduced by Sih [11] for cracks in homogeneous materials. In the following the generalized strain energy density factor Σ for a crack terminating at the interface is presented.

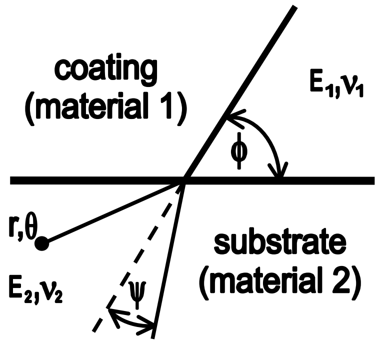

Fig. 1. A crack terminating at the interface at the angle ϕ , the direction of the crack growth in the substrate is given by the angle ψ.

Strain energy density.

The Strain energy density w is given by

$$w = \frac{dW}{dV} = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \tag{3}$$

where $\sigma_{ij} = \sigma^{I}_{ij} + \sigma^{II}_{ij}$ and $\varepsilon_{ij} = \varepsilon^{I}_{ij} + \varepsilon^{II}_{ij}$ the are corresponding components for mode I and II of the stress and strain, respectively, in a volume element dV [10]. Reduced to plane problems, w can be written as follows (our considerations are limited to the material 2, i.e., the substrate)

$$\frac{dW}{dV} = \frac{1}{8\mu_2}\left[k\left(\sigma_x + \sigma_y\right)^2 + \left(\sigma_x - \sigma_y\right)^2 + 4\sigma_{xy}^{\ 2}\right] \tag{4}$$

For plane strain and $k = (1\text{-}2\nu_2)$, μ_2 is the shear modulus and ν_2 is the Poisson's ratio of the substrate.

Distribution of the strain energy density. The expression for strain energy density at the tip of the crack can now be easily derived from formulas 2 and 4. It holds that

$$\frac{dW}{dV} = \left[A_{11}H_I^{\ 2} + 2A_{12}H_I H_{II} + A_{22}H_{II}^{\ 2}\right] r^{\frac{3}{2}-p} \tag{5}$$

where A_{11} , A_{12} and A_{22} are functions of the polar angle θ , the composite parameters α, β, and the angle ϕ between the crack and the interface. H_I and H_{II} are the values of the stress intensity factors corresponding to normal and shear modes of loading.

Strain energy density factor. Let us write expression (5) in the form

$$w = \frac{dW}{dV} = \frac{\Sigma(\alpha,\beta,\theta,\Phi,r)}{r}, \tag{6}$$

where
$$\Sigma = \left(A_{11} H_I^{\,2} + 2A_{12} H_I H_{II} + A_{22} H_{II}^{\,2} \right) r^{\frac{1}{2}-p} \tag{7}$$

The function Σ generalizes the strain energy density factor for cracks terminating at the interface and is used in the following to formulate the stability conditions.

Unlike the homogeneous energy density factor S, the generalized strain energy density factor Σ depends on the distance r from the tip of the crack, see Eq. (6).

Formulation of the stability criterion

A theory based on the concept of the strain energy density factor S formulated by Sih [11] for a crack in homogeneous materials (i.e. for power of singularity $p = 1/2$) proceeds from two fundamental hypotheses about the extension of a crack:

the crack initiation will start in the radial direction along which the strain energy density S is a minimum, and the critical value of the strain energy density $S = S_{cr}$ governs the onset of the crack propagation.

Note that S_{cr} is a material constant and in special cases can be related to K_{IC}, the fracture toughness of the material.

In the same way, for a crack with its tip at the interface (i.e., for a power of singularity $p \neq 1/2$), a generalized strain energy density approach can be formulated, where $S = \Sigma$, the generalized strain energy density factor. Again, the first hypothesis can be used to predict the direction propagation of the crack into the substrate (see Fig.1) and the second hypothesis determines the onset of the crack propagation. Moreover, if we assume that the presence of the interface influences the propagation of the crack into the substrate only quantitatively and that the mechanism of the crack propagation is the same, it holds that

$$S_{cr} = \Sigma_{cr} r^{\frac{1}{2}-p}, \tag{8}$$

where r is the unknown distance from the tip of the crack at which the criterion is applied. The stability condition then has the form

$$\Sigma < \Sigma_{cr}. \tag{9}$$

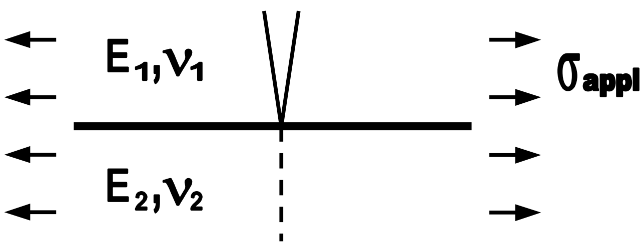

Fig. 2. A crack terminating perpendicularly at the interface. The applied stress σ_{appl} is oriented parallel to the interface. This configuration corresponds to the normal mode of loading.

Stability criterion for the normal mode of loading. If a crack terminating normally at the interface and the applied stress is oriented parallel to the interface, see Fig.2., the stress state around the crack tip corresponds to the normal mode of loading and $H_{II} = 0$. The crack will grow into the substrate perpendicularly to the interface. Moreover only one real root is found for the power of singularities *p* for the usual combinations of materials.

Then

$$\Sigma = A_{11} H_I^{\,2} r^{\frac{1}{2}-p} , \tag{10}$$

where

$$\begin{aligned} A_{11} = \frac{\lambda^2}{4\mu_2} [4k\left(\cos\left(\lambda\text{-}1\right)\theta\right)^2 + \left(g_R \cos\left(\lambda\text{-}1\right)\theta + \left(\lambda\text{-}1\right)\cos\left(\lambda-3\right)\theta\right)^2 + \\ + \left(g_R \sin\left(\lambda\text{-}1\right)\theta + \left(\lambda\text{-}1\right)\sin\left(\lambda-3\right)\theta\right)^2] \end{aligned} , \tag{11}$$

$$\lambda = p - 1,$$

$$g_R = \lambda - \cos\lambda\pi - \frac{\beta[\alpha + 2\lambda - (1 + 2\alpha - 4\alpha\lambda^2)\cos\lambda\pi + (1+\alpha)\cos 2\lambda\pi]}{D(\lambda)},$$

$$D(\lambda) = 1 + 2\alpha + 2\alpha^2 - 2(\alpha + \alpha^2)\cos\lambda\pi - 4\alpha^2\lambda^2 ,$$

It follows from Eq. (9) that the crack will propagate if

$$H_I < H_{IC} , \tag{12}$$

where H_{IC} is the critical value of the generalized stress intensity factor for normal loading,

$$H_{IC} = \left(\frac{1\text{-}2\nu}{(1\text{-}p)^2 \left(4\left(1\text{-}2\nu\right) + \left(g_R \text{-} p\right)^2\right)} \right)^{\frac{1}{2}} r^{p-\frac{1}{2}} K_{IC} \tag{13}$$

Application of the stability condition (9) makes it possible to estimate the critical applied stress σ_{cr}. If the value of the applied stress $\sigma_{appl} > \sigma_{cr}$ the crack will grow into the substrate. The value of σ_{cr} depends on the geometry and the boundary conditions, on the composite parameters α and β, and on the fracture toughness K_{IC} of the substrate. Unlike the case for a homogeneous material, for this case the condition of stability, and thus the critical applied stress σ_{cr} , depends on the distance *r* for which the condition is applied. A numerical example of this calculation is given in the next chapter.

NUMERICAL EXAMPLE

Bodies with protective layers correspond to combinations of two materials, and everything derived above for cracks in bi-materials is generally valid. The coating thickness is usually small compared to the thickness of the substrate, and in the following the numerical model shown in Fig.3 will be used. We supposed a single crack in the coating which propagates perpendicularly to the surface, and we estimate the applied critical stress σ_{cr} . Knowing the value of σ_{cr} makes it possible to decide if the crack will stop at the interface or penetrate into the substrate. The elastic constants of the coating and the interface are given in Table 1. Calculation of the critical stress σ_{cr} corresponding to the proposed criterion consists of the following steps:

Estimate α and β, and the value of the power of singularity p, and express the stress components and the expression for Σ .

Evaluate the generalized stress intensity factor H_I for the given geometry and material parameters, see Table 1.

Apply the stability conditions and estimate the critical applied stress,

$$\sigma_{cr}=\frac{\left(\dfrac{1-2\nu}{(1-p)^2\left(4(1-2\nu)+(g_R-p)^2\right)}\right)^{\frac{1}{2}}}{H_I\,(1\,\mathrm{MPa})}\,r^{p-\frac{1}{2}}K_{IC}\,, \tag{14}$$

where K_{IC} is the fracture toughness of the substrate (material 2).

Table 1. The composite parameters α and β, the corresponding value of the power of singularity p, and the generalized stress intensity factor H_I . The values presented correspond to a tension specimen with a protective layer (Fig.3), the applied stress σ_{appl} = 1MPa.

E_1/E_2	α	β	p	H_I [Mpa.m^p]
0.2	-0.286	0.2	0.3662	13.90e-2
0.5	-0.179	0.5	0.4339	9.02e-2
1.0	0.000	1.0	0.5000	6.28e-2
2.0	0.357	2.0	0.5745	4.49e-2
5.0	1.429	5.0	0.6789	3.22e-2

Calculations of H_I values have been performed by finite element system ANSYS [1].

The value of the distance r at which the criterion is applied influences the resulting values of the critical stress, but the dependence is not to strong, see Table 2, where results for two possible values of r are presented. In the first case r was set to the value r_p corresponding to the size of the plastic zone in the substrate as calculated by linear elastic fracture mechanics under plane strain conditions. For a cleavage type of fracture, the distance r can be related to the grain size of material, see Table 2, where the critical stresses for $d = 100\mu m$ are presented. Moreover, for comparison, the critical stress calculated by another criterion based on the average stress calculated across the distance $r = d$ from the crack tip [6], is presented. It can be concluded that for the current materials for which the ratio is $0.2 < E_1/E_2 < 5$ all approaches give comparable results.

The resultant calculations are presented in Table 2, where the values of the critical stress σ_{cr} for different values of the coating/substrate ratio E_1/E_2 are presented.

Table 2. Values of the critical stress σ_{cr} corresponding to a tension specimen (Fig.3). $\sigma^{(1)}_{cr}$ corresponds to Eq.(17) with $r = r_p$, $\sigma^{(2)}_{cr}$ corresponds to Eq.(17) with $r = d = 100\mu$, and $\sigma^{(3)}_{cr}$ is taken from reference [6].

E_1/E_2	$\sigma^{(1)}_{cr}$ [Mpa]	$\sigma^{(2)}_{cr}$ [Mpa]	$\sigma^{(3)}_{cr}$ [Mpa]
0.2	50.7	38.5	46.9
0.5	61.3	57.0	63.8
1.0	79.6	79.6	79.6
2.0	109.7	110.2	92.4
5.0	166.6	158.8	94.9

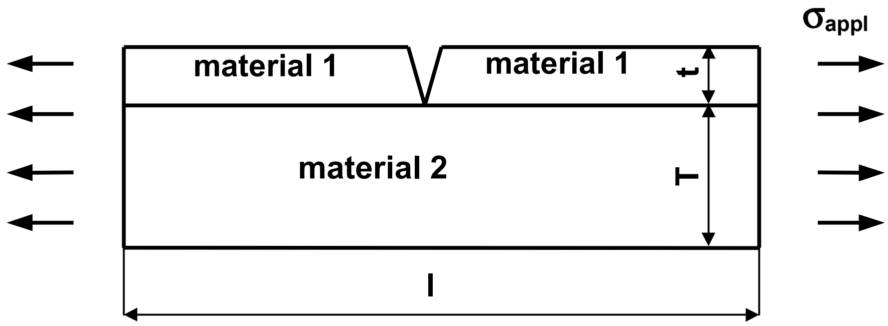

Fig. 3. Dimensions and geometry of a tension specimen with a protective surface layer. T = 15 mm, t = 1 mm, l = 30 mm.

CONCLUSIONS

A fracture criterion that deals with the initiation of a crack which propagates to fracture from of a crack terminating at an interface has been suggested. The problem was studied under assumptions of linear elastic fracture mechanics. The criterion of stability formulated is based on application of the strain energy density concept as originally formulated by Sih for cracks in homogeneous materials. To this aim general formulas that express the generalized strain energy density factor for a crack terminating at an interface have been derived. It was assumed that the crack initiation starts in the radial direction in which the generalized strain energy density factor Σ is a minimum and that the critical value $\Sigma = \Sigma_{cr}$ governs the onset of crack propagation across the interface. Special attention has been devoted to the case of a crack perpendicular to the interface. The critical applied stress σ_{cr} for a tension specimen with a protective surface layer has been calculated as an example. The critical stress to failure the substrate depends on the composite parameters α and β and on the value of the fracture toughness of the substrate. The existence of cracks in the surface brittle coating on a tough substrate may substantially decrease the critical failure stress of that substrate.

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