

## PYRAMIDAL MODEL OF INTERGRANULAR CRACK FRONT

P. ŠANDERA<sup>1)</sup>, J. POKLUDA<sup>1)</sup> AND J. HORNÍKOVÁ<sup>2)</sup>

*<sup>1)</sup>Institute of Physical Engineering, <sup>2)</sup>Institute of Mechanics of Solids,  
Brno University of Technology, Faculty of Mechanical Engineering, Technická 2,  
61669 Brno, Czech Republic*

### ABSTRACT

The simple pyramidal-like model is introduced as suitable approximation of the real geometry of intergranular crack fronts in metallic materials. The extrinsic component of the measured fracture toughness  $K_{Ic}$  can be estimated by means of this model. Standardly measured materials data as the yield stress, the mean grain size and the linear surface roughness are needed for application of this simple approach. Values of the global effective stress intensity factors received in the framework of the pyramidal model agree well with those obtained for complicated real-like intergranular geometries constructed on the bases of 3D Voronoi tessellation.

### KEYWORDS

Effective stress intensity factor, intergranular crack front, shielding effect, pyramidal model.

### INTRODUCTION

The fracture toughness of materials can be assumed to be composed of two basic components – intrinsic and extrinsic toughness [1]. The first one is associated with the inherent materials resistance against the crack growth and can be improved by increasing both the strength of interatomic bonds and the matrix plasticity by changing chemical composition or application of appropriate heat treatments. The latter one is connected with factors and processes reducing the crack driving force as, e.g., the tortuous crack tip geometry and the related surface roughness in the crack wake (i.e. the roughness induced crack tip shielding), bridging of crack surfaces by secondary phases or strengthening fibres and phase transformations of metastable particles. It leads to an apparent increase in the intrinsic resistance against the crack growth caused, in fact, by an extrinsic factor.

The main aim of this paper is to present a simple pyramidal model of the intergranular crack front enabling to assess the global effective stress intensity factor. Conditions of both the small-scale yielding and plane strain are assumed to be particularly fulfilled in this analysis.

## CALCULATION OF THE EFFECTIVE STRESS INTENSITY FACTOR BY MEANS THE PYRAMIDAL MODEL

The intergranular microtortuosity induces a local mixed mode loading at the crack front even in case of a pure remote mode I loading. Hence, at the real 3D crack front acts the local mixed mode 1 + 2 + 3 described by local stress intensity factors  $k_1$ ,  $k_2$ , and  $k_3$ . A further contribution to the shielding effect is caused by the fact that the tortuous crack front is longer than the straight one (of the same projective length). A simultaneous consideration of both the above mentioned shielding contributions [2] leads to the following expression for the global normalised effective stress intensity factor:

$$k_{eff}^2 = \frac{\frac{1}{B} \int_0^B k_{eff}^2 dz}{R_L} = \frac{k^{*2}}{R_L}, \quad (1)$$

where  $B$  is the thickness of the specimen,  $z$  is the Cartesian co-ordinate along the thickness,  $R_L$  is the surface roughness and

$$k_{eff}^2 = k_1^2 + k_2^2 + \frac{1}{1-\nu} k_3^2,$$

$\nu$  is the Poisson ratio.

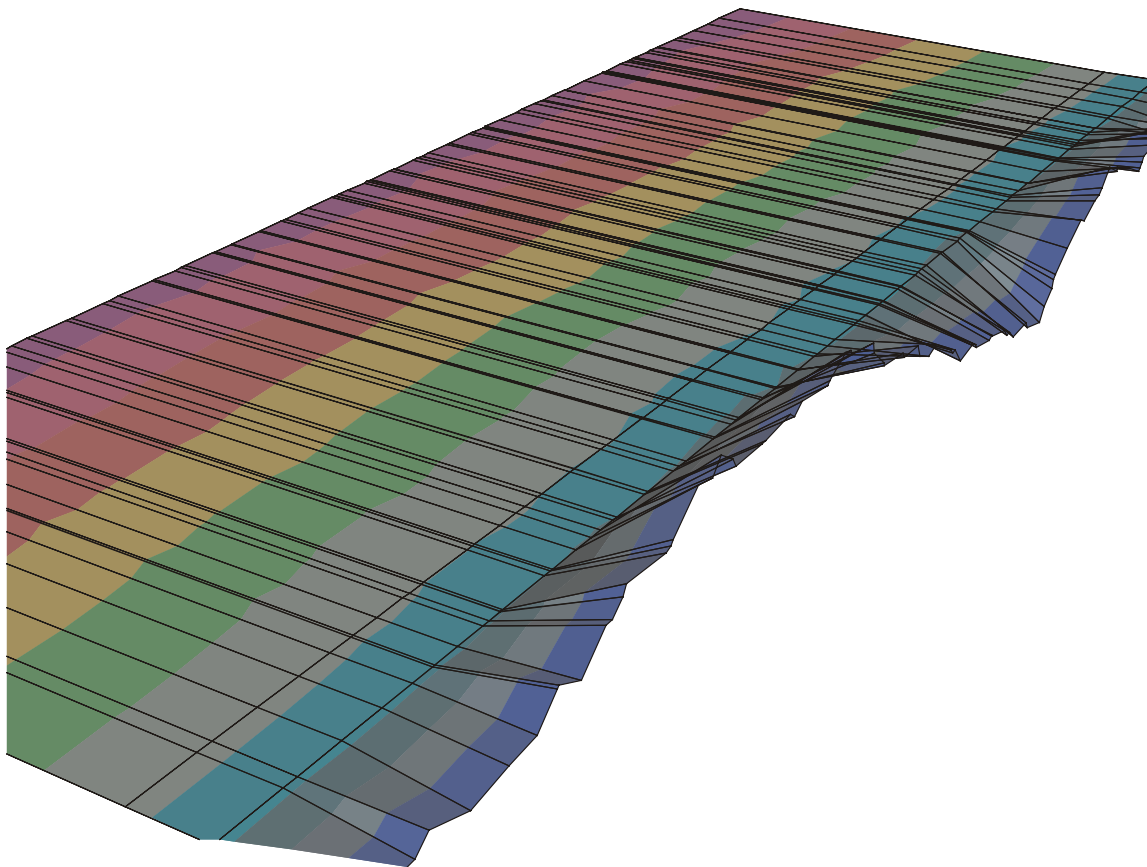


Fig. 1 Computer model of the intergranular crack front based on 3D Voronoi tessellation.

The intergranular subcritical crack developing at the front of a fatigue pre-crack during the fracture toughness test can be simulated by a narrow tortuous band following the grain boundary network in the related 3D Voronoi tessellation (see Fig. 1) [3]. The shielding factor for this complicated geometry corresponds well to a much more simple periodic pyramid-like geometry – see Fig. 2. Each oblique segment at the front of this band can be understood to be associated with one grain of the regular grain boundary network related to the real one by the mean grain size  $d_m$ . The pyramidal band can be simply characterised by angles  $\Theta_m$  and  $\Phi$  with respect to the macroscopic fatigue crack plane. The value  $\Phi = \pi/4$  corresponds to the roughness  $R_L = 1,41$  of the band front typical for intergranular crack surfaces in metallic materials independently on the mean grain size [4,5]. The maximal angle  $\Theta_m$  at the end of the oblique segment is related to  $\Phi$  as

$$d_m \cdot \tan \Phi = r_p \cdot \tan \Theta_m, \quad (2)$$

where  $r_p$  is the plastic zone size at the moment of unstable fracture initiation:

$$r_p \approx \frac{1}{3\pi} \left( \frac{K_{Ic}}{\sigma_y} \right)^2.$$

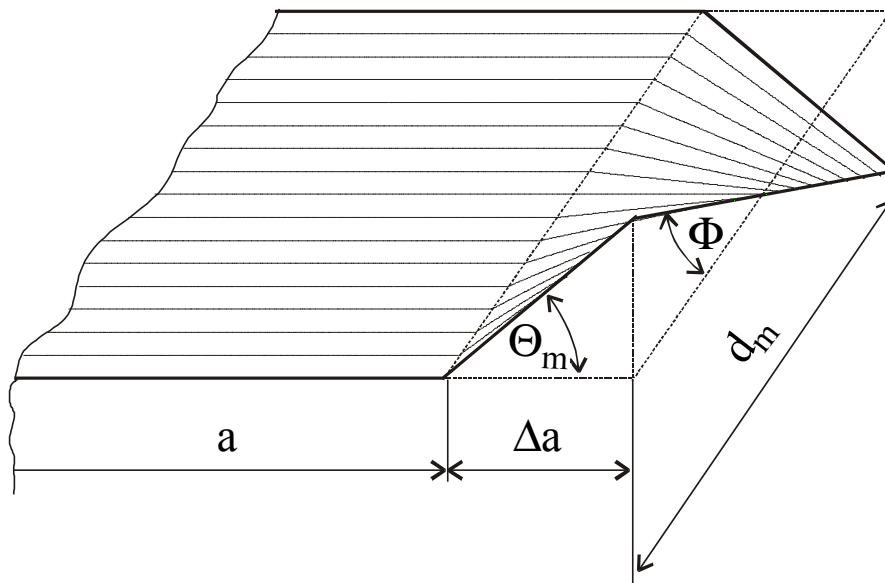


Fig. 2 Element of the pyramidal approximation of the intergranular crack front band.

The stable intergranular crack develops within the process zone of size  $\Delta a$  proportional to that of the plastic zone of the *a priori* crack. The one half of the plastic zone size  $r_p$  can be accepted as a plausible measure of  $\Delta a$ , i.e.  $\Delta a = r_p / 2$ .

Then, the effective stress intensity factor  $k_{effg}$  can be calculated by using approximate analytical expressions for local stress intensity factors (normalised to the remote  $K_I$  factor)

$$\begin{aligned}
k_1 &= \cos^6\left(\frac{\Theta}{2}\right) \cdot \left[ 2\nu \sin^2 \Phi + \cos^2\left(\frac{\Theta}{2}\right) \cos^2 \Phi \right] + \\
&\quad + \sin^2\left(\frac{\Theta}{2}\right) \cos^4\left(\frac{\Theta}{2}\right) \left[ 2\nu \sin^2 \Phi + 3 \cos^2\left(\frac{\Theta}{2}\right) \cos^2 \Phi \right], \\
k_2 &= \sin\left(\frac{\Theta}{2}\right) \cos^2\left(\frac{\Theta}{2}\right), \\
k_3 &= \cos^6\left(\frac{\Theta}{2}\right) \cdot \sin \Phi \cos \Phi \left[ \cos^2\left(\frac{\Theta}{2}\right) - 2\nu \right] + \\
&\quad + \sin^2\left(\frac{\Theta}{2}\right) \cos^4\left(\frac{\Theta}{2}\right) \sin \Phi \cos \Phi \left[ 3 \cos^2\left(\frac{\Theta}{2}\right) - 2\nu \right].
\end{aligned} \tag{3}$$

Eqs. (3), similar to those reported in [6], hold reasonably well for  $\Delta a \ll a$ , where  $a$  is the length of an *a priori* fatigue crack. By using the energy criterion for the plain strain, the effective stress intensity factor  $k_{effg}$  can be computed as

$$k_{effg}^2 = \frac{k^{*2}}{R_L} = \frac{1}{2R_L \Theta_m} \int_{-\Theta_m}^{\Theta_m} \left( k_1^2 + k_2^2 + \frac{k_3^2}{1-\nu} \right) d\Theta. \tag{4}$$

The dependence  $k^*$  vs.  $d_m$  for various values  $r_p$  is shown in Fig. 3.

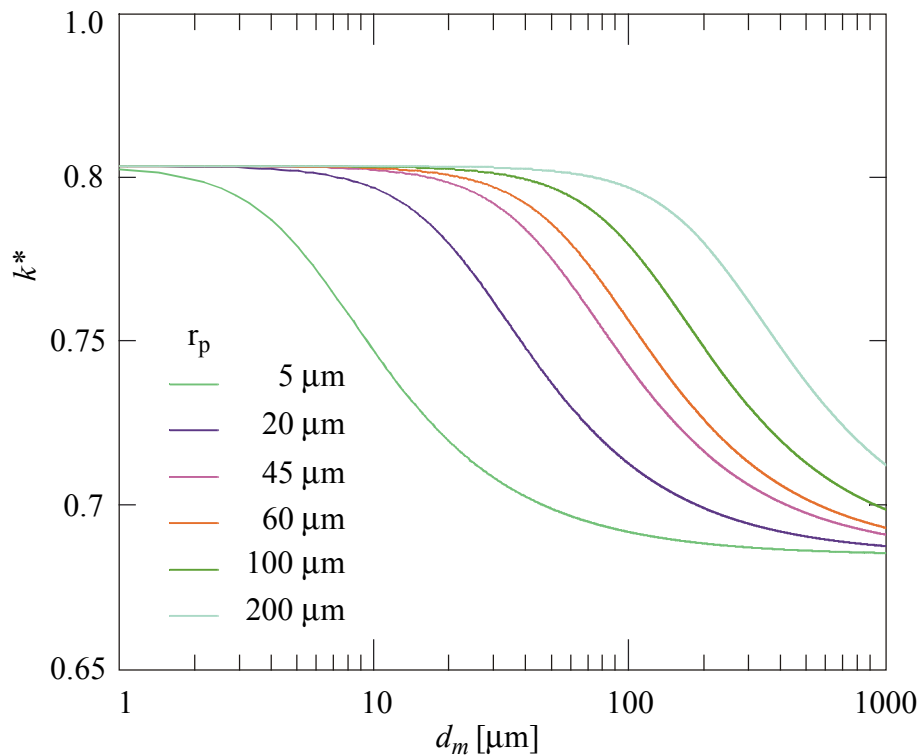


Fig. 3 Averaged normalised local stress intensity factor as function of mean grain size for various plastic zone sizes.

The surface roughness for intergranular cracks varies in a relatively close range of  $R_L \in <1.4; 1.5>$  [4,5]. Thus, by knowing the values of the yield stress  $\sigma_y$ , mean grain size  $d_m$  and the measured fracture toughness  $K_{Ic}$  for the particular material, the global effective stress intensity factor  $k_{effg}$  can be estimated according to equation (1) and Fig. 3. Values of the global effective stress intensity factors received in the framework of the pyramidal model agree well with those obtained for complicated real-like intergranular geometries constructed on the bases 3D Voronoi tessellation [7,8].

## CONCLUSION

Intergranular crack fronts are accompanied by a low global effective stress intensity factor leading to a high measured fracture toughness  $K_{Ic}$ . However, the intrinsic component of the fracture toughness, expressing much better the inherent resistance of the material against the crack growth initiation, remains to be small. Therefore, overestimation of materials quality by too much respecting the high values of measured fracture toughness  $K_{Ic}$  could be very dangerous. Hence, procedures separating the extrinsic component of fracture toughness from the experimental data are of a great practical importance.

In this paper, an approximate method for the estimation of the extrinsic component of fracture toughness is presented based on a simple pyramidal model of the intergranular crack front. Only standard materials data (yield stress, mean grain size, linear surface roughness) are needed in order to apply this simple approach.

## ACKNOWLEDGEMENT

The work was supported by the European agency COST (Action 517) and by the Ministry of Education and Youth of the Czech Republic under the Grant No. OC 517.30 (1998).

## REFERENCES

- [1] RITCHIE R.O.: Mat. Sci. Eng. A103, 1988, 15.
- [2] POKLUDA J.: In: FRACTOGRAPHY 2000, ed. L. Parilák. Emilena Bratislava 2000, p. 82.
- [3] HORNÍKOVÁ J.: On the Linear Fracture Mechanics for Cracks with Microscopically Tortuous Front. PhD. Thesis. Brno UT 2000.
- [4] POKLUDA J., SAXL I., ŠANDERA P., PONÍŽIL P., MATOUŠEK M., PODRÁBSKÝ T. and HORNÍKOVÁ J.: In: Advances in Mechanical Behaviour, Plasticity and Damage - Euromat 2000, eds.: D.Miannay, P.Costa, D.Francois, A.Pineau. ELSEVIER 2000, p. 449
- [5] SAXL I., SÜLLEIOVÁ K. and PONÍŽIL P.: In: FRACTOGRAPHY 2000, ed. L. Parilák. Emilena Bratislava 2000, p. 94.
- [6] FABER K.T. and EVANS A.G.: Acta Metall. 31, 1983, 565.
- [7] HORNÍKOVÁ J., ŠANDERA P. and POKLUDA J., this proceedings.
- [8] POKLUDA J., In: VII Summer School of Fracture Mechanics, Pokrzywna 2001, (in print).